
Being alive OR Being dead
Schrödinger's Cat

Fundamentos Básicos de la Computación Cuántica

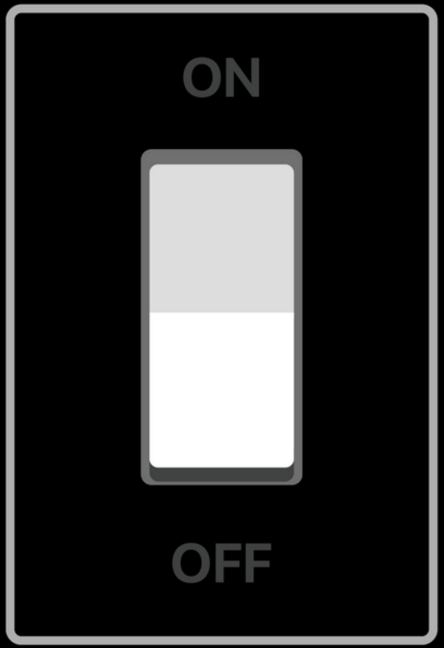
QISKIT FALL FESLT 2025

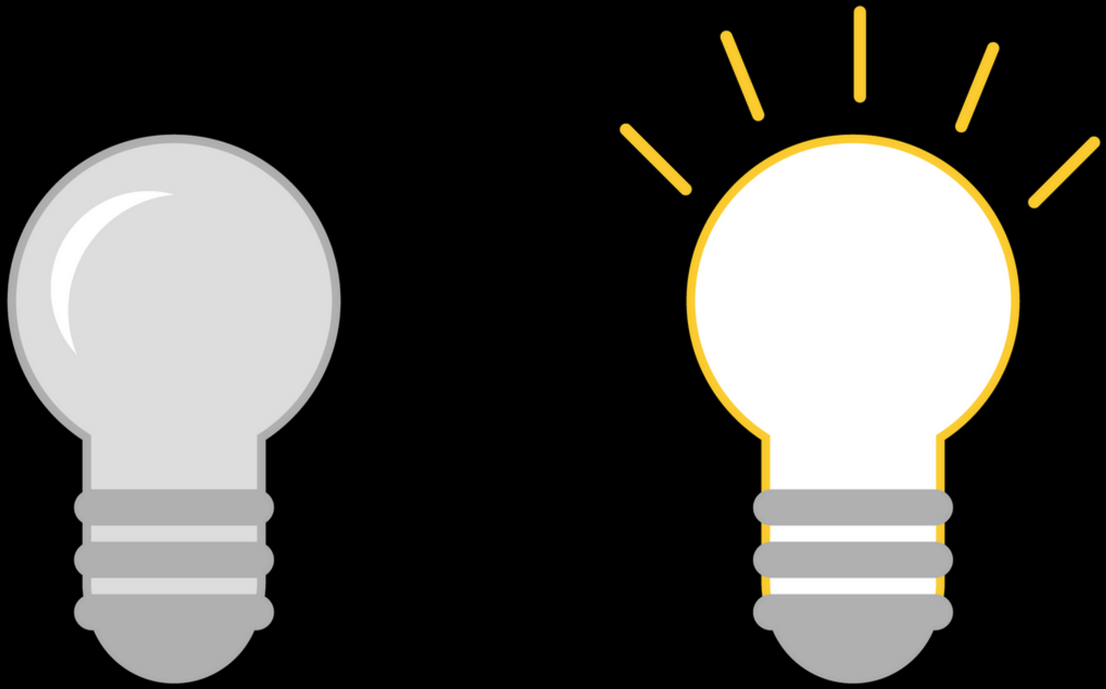
Marcela Herrera

Universidad del Valle

From bits to qubits

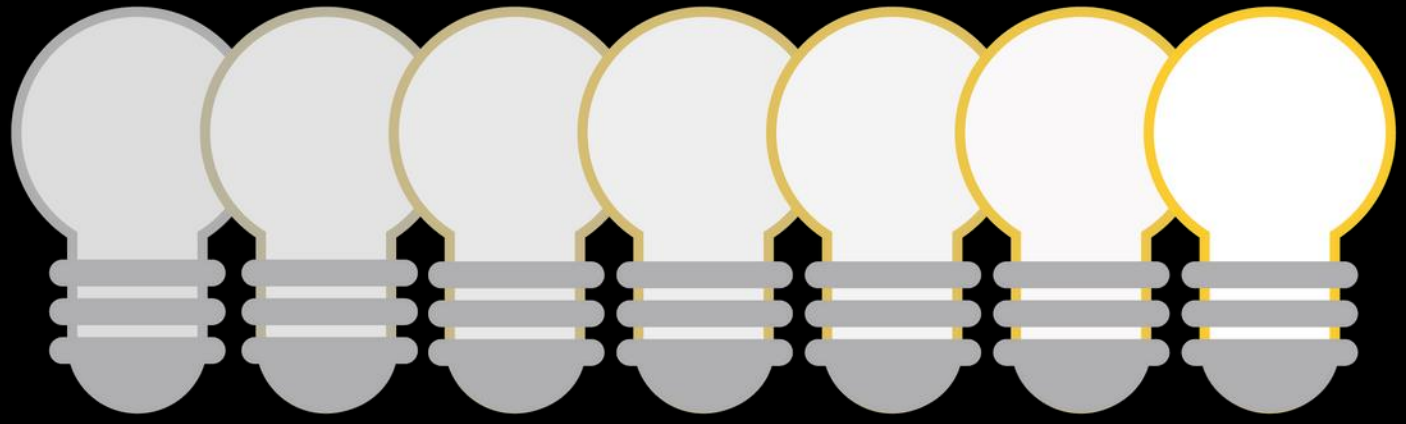
bit





What is a qubit?

Qubit: Quantum bit



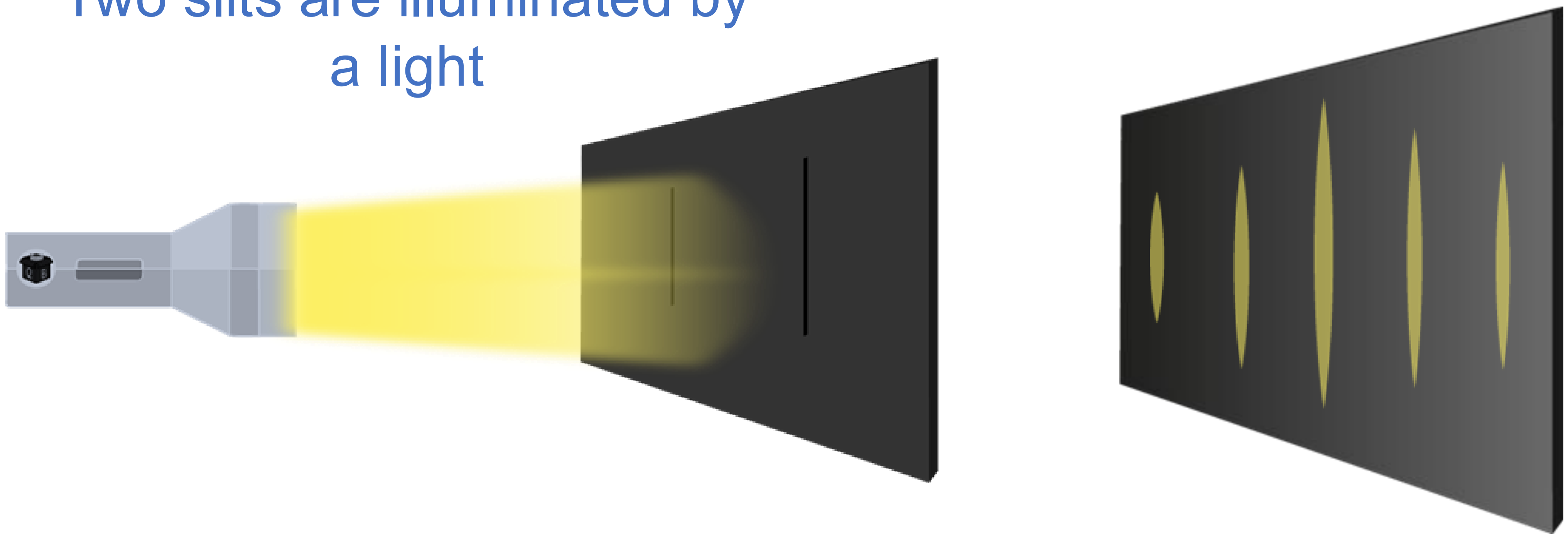
State superposition

Awake **Sleep**

Superposition

Wave-particle duality

Two slits are illuminated by
a light



@quantum.box

BITS

0 o 1

2 BITS

01

3 BITS

011

QUBITS

0 y 1

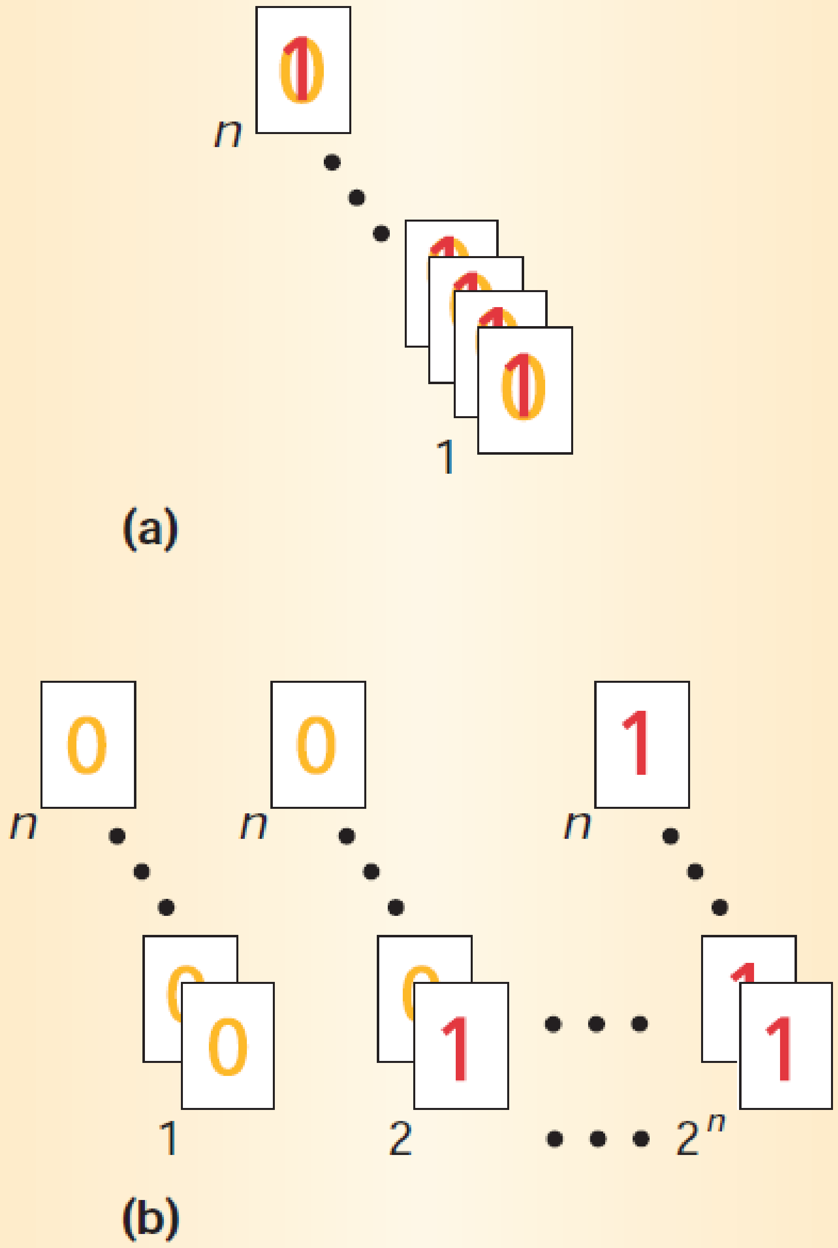
2 QUBITS

00 01 10 11

3 QUBITS

000 001 010 100

011 101 110 111



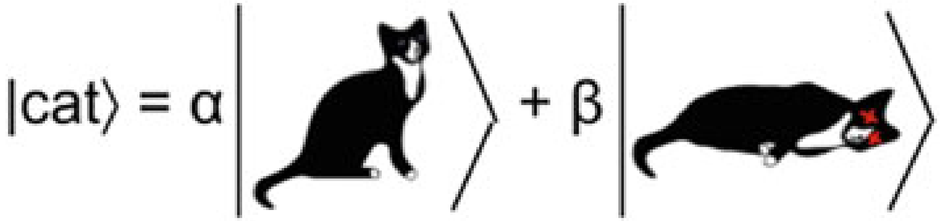
To achieve the same degree of parallelism as **(a)** **300** quantum processors(**n = 300**), we would need

(b) $2^{300} \approx 2{,}04 \times 10^{90}$ classical processors

Since 2^{300} is more than the number of particles in the universe, to say that quantum computing enables an astronomical increase in parallelism is obviously an understatement

"Beyond Bits: The Future of Quantum Information Processing" in Computer, vol. 33, no. 01, pp. 38-45, 2000.

Quantum state



$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

with α and β called the amplitudes of the states. Amplitudes are generally complex numbers

$$|\alpha|^2 + |\beta|^2 = 1$$

This is called a **normalization** rule

Book: Quantum Computing for the Quantum Curious

Example

1. The quantum state of a spinning coin can be written as a superposition of heads and tails. Using heads as $|1\rangle$ and tails as $|0\rangle$, the quantum state of the coin is

$$|\text{coin}\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |0\rangle\right).$$

   What is the probability of getting heads?

The amplitude of $|1\rangle$ is $\beta = 1/\sqrt{2}$, so $|\beta|^2 = \left(1/\sqrt{2}\right)^2 = 1/2$. So the probability is 0.5, or 50%.

Book: Quantum Computing for the Quantum Curious

Example

2. A weighted coin has twice the probability of landing on heads vs. tails. What is the state of the coin in “ket” notation?

$$P_{\text{heads}} + P_{\text{tails}} = 1 \quad \text{(Normalization Condition)}$$

$$P_{\text{heads}} = 2P_{\text{tails}} \quad \text{(Statement in Example)}$$

$$\rightarrow P_{\text{tails}} = \frac{1}{3} = \alpha^2$$

$$\rightarrow P_{\text{heads}} = \frac{2}{3} = \beta^2$$

$$\rightarrow \alpha = \sqrt{\frac{1}{3}},\ \beta = \sqrt{\frac{2}{3}}$$

$$|\text{coin}\rangle = \sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle.$$

Book: Quantum Computing for the Quantum Curious

Example

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\pi/6}|1\rangle\right)$$

Is this state normalized?

$$|\alpha|^2 + |\beta|^2 = 1$$

Normalization rule

$$\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

$$\left|\frac{e^{i\pi/6}}{\sqrt{2}}\right|^2 = \frac{e^{i\pi/6}}{\sqrt{2}}\frac{e^{-i\pi/6}}{\sqrt{2}} = \frac{e^0}{2} = \frac{1}{2}$$

Example

$$\frac{1}{\sqrt{3}}\left(\sqrt{2}|0\rangle + |1\rangle\right)$$

Is this state normalized?

$$|\alpha|^2 + |\beta|^2 = 1$$

Normalization rule

$$\left|\sqrt{\frac{2}{3}}\right|^2 = \frac{2}{3}$$

$$\left|\frac{1}{\sqrt{3}}\right|^2 = \frac{1}{3}$$

Measuring a qubit does not produce an average of $|0\rangle$ and $|1\rangle$: the qubit collapses to one definite state. A single measurement cannot reveal α or β; many identical qubits are needed to observe how often outcomes collapse to $|0\rangle$ or $|1\rangle$

Matrix representation

When writing a single qubit in a superposition $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
In matrix representation, a qubit is written as a two-dimensional vector where the amplitudes are the components of the vector

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

The states $|0\rangle$ and $|1\rangle$ are usually represented as

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Book: Quantum Computing for the Quantum Curious

Changing a qubit's state through a physical action mathematically corresponds to multiplying the qubit vector $|\psi\rangle$ by some **unitary matrix** *U* so that after the operation the state is now

$$|\psi'\rangle = U|\psi\rangle.$$

A matrix *U* is unitary if the matrix product of *U* and its conjugate transpose $U^\dagger$ (called *U*-dagger) multiply to give the identity matrix:

$$UU^\dagger = U^\dagger U = \mathbb{I}$$

Book: Quantum Computing for the Quantum Curious

Review: matrix multiplication

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax + by + cz \\ dx + ey + fx \\ gx + hy + iz \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}$$

Review: matrix multiplication

$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\mathrm{w}\\ x\\ y\\ z\end{pmatrix} = \begin{pmatrix}\mathrm{w}\\ x\\ y\\ z\end{pmatrix}$$

$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\mathrm{w}\\ x\\ y\\ z\end{pmatrix} = \begin{pmatrix}\mathrm{w}\\ \mathrm{y}\\ \mathrm{x}\\ z\end{pmatrix}$$

Review: transpose of a matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad A^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \qquad A^T = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$$

Example

What is the conjugate transpose of the following matrix?

$$A = \begin{pmatrix} 1 & i \\ 1 & i \end{pmatrix}$$

$$A^{\dagger} = \begin{pmatrix} 1 & 1 \\ -i & -i \end{pmatrix}$$

Book: Quantum Computing for the Quantum Curious

Example

Is the matrix A unitary? $UU^{\dagger} = U^{\dagger}U = \mathbb{I}$

$$A = \begin{pmatrix} 1 & i \\ 1 & i \end{pmatrix}$$

$$AA^{\dagger} = \begin{pmatrix} 1 & i \\ 1 & i \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -i & -i \end{pmatrix}$$

$$A^{\dagger} = \begin{pmatrix} 1 & 1 \\ -i & -i \end{pmatrix}$$

$$= 2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Book: Quantum Computing for the Quantum Curious

Example

The operator X is unitary?

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad X^\dagger = X^T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$X^\dagger X = XX = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Book: Quantum Computing for the Quantum Curious

Example

What is the result of applying the unitary operator X onto a $|0\rangle$ state qubit?

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

$$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle.$$

Book: Quantum Computing for the Quantum Curious

Operations on one classical bit

Identity	$0 \longrightarrow 0$ $1 \longrightarrow 1$	$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$	$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$
Negation	$0 \longrightarrow 1$ $1 \longrightarrow 0$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$
Constant-0	$0 \longrightarrow 0$ $1 \longrightarrow 0$	$\begin{pmatrix}1 & 1\\0 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$	$\begin{pmatrix}1 & 1\\0 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$
Constant-1	$0 \longrightarrow 1$ $1 \longrightarrow 1$	$\begin{pmatrix}0 & 0\\1 & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}1 & 1\\0 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$

Reversible computing

- Reversible means given the operation and output value, you can find the input values

Identity and Negation are reversible

$$\begin{array}{ccc} 0 & \longrightarrow & 0 \\ 1 & \longrightarrow & 1 \end{array} \qquad \begin{array}{ccc} 0 & \longrightarrow & 1 \\ 1 & \longrightarrow & 0 \end{array}$$

Constant-0 and Constant-1 are not reversible

$$\begin{array}{ccc} 0 & \longrightarrow & 0 \\ 1 & \longrightarrow & 0 \end{array} \qquad \begin{array}{ccc} 0 & \longrightarrow & 1 \\ 1 & \longrightarrow & 1 \end{array}$$

- Quantum computers use only reversible operation

Check Your Understanding

Assume a flipped coin can be measured as either heads (H) or tails (T).

a) If the coin is in a normalized state $\frac{1}{\sqrt{10}}|H\rangle + \frac{3}{\sqrt{10}}|\mathrm{T}\rangle$, what is the probability that the coin will be tails?

b) During a flip, the coin is in a state $\frac{1}{3}|H\rangle + \frac{2}{3}|\mathrm{T}\rangle$ .Is this state normalized?

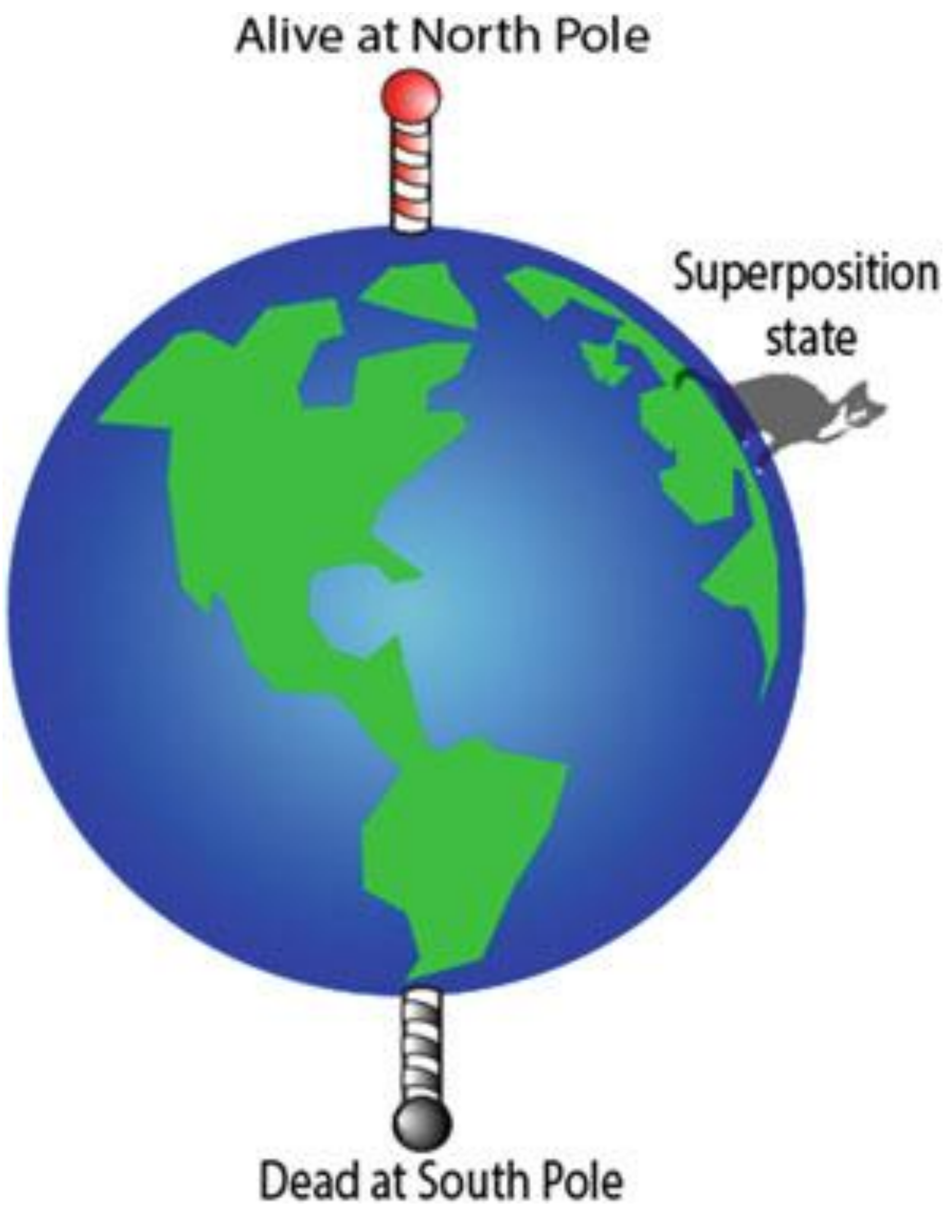
c) A machine is built to flip coins and put them into a state $\frac{1}{3}|H\rangle + \frac{\sqrt{3}}{2}|\mathrm{T}\rangle$ when flipped. If 100 coins are flipped, how many coins should land on tails?

d) What is the matrix product of the matrix

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

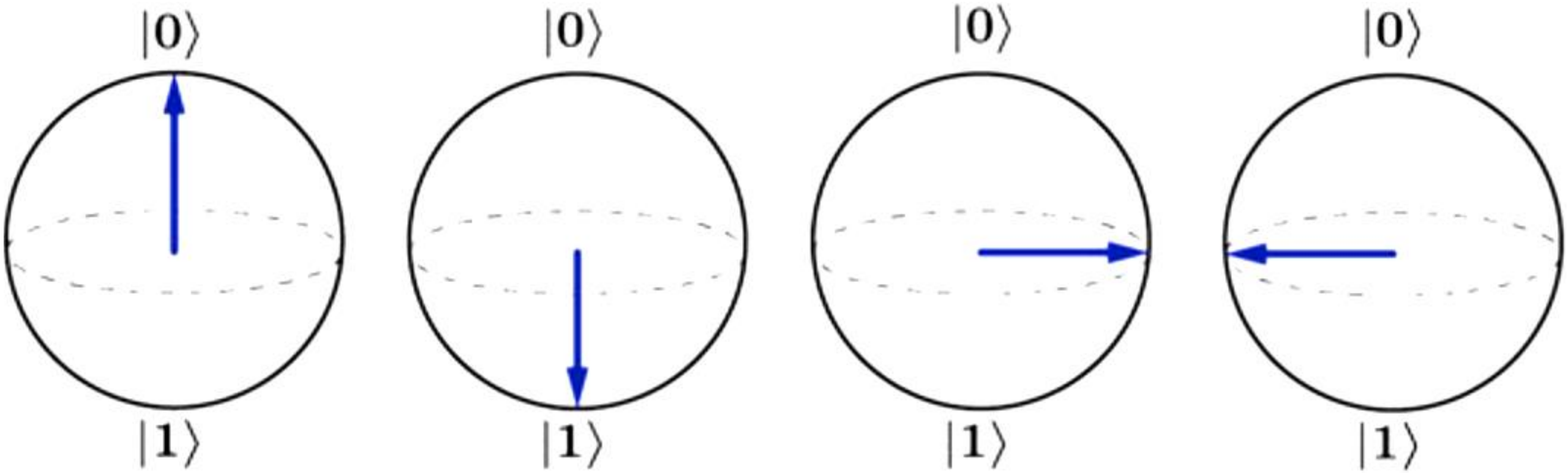
and a qubit in the general state $\alpha|0\rangle + \beta|1\rangle$

Bloch sphere



Book: Quantum Computing for the Quantum Curious

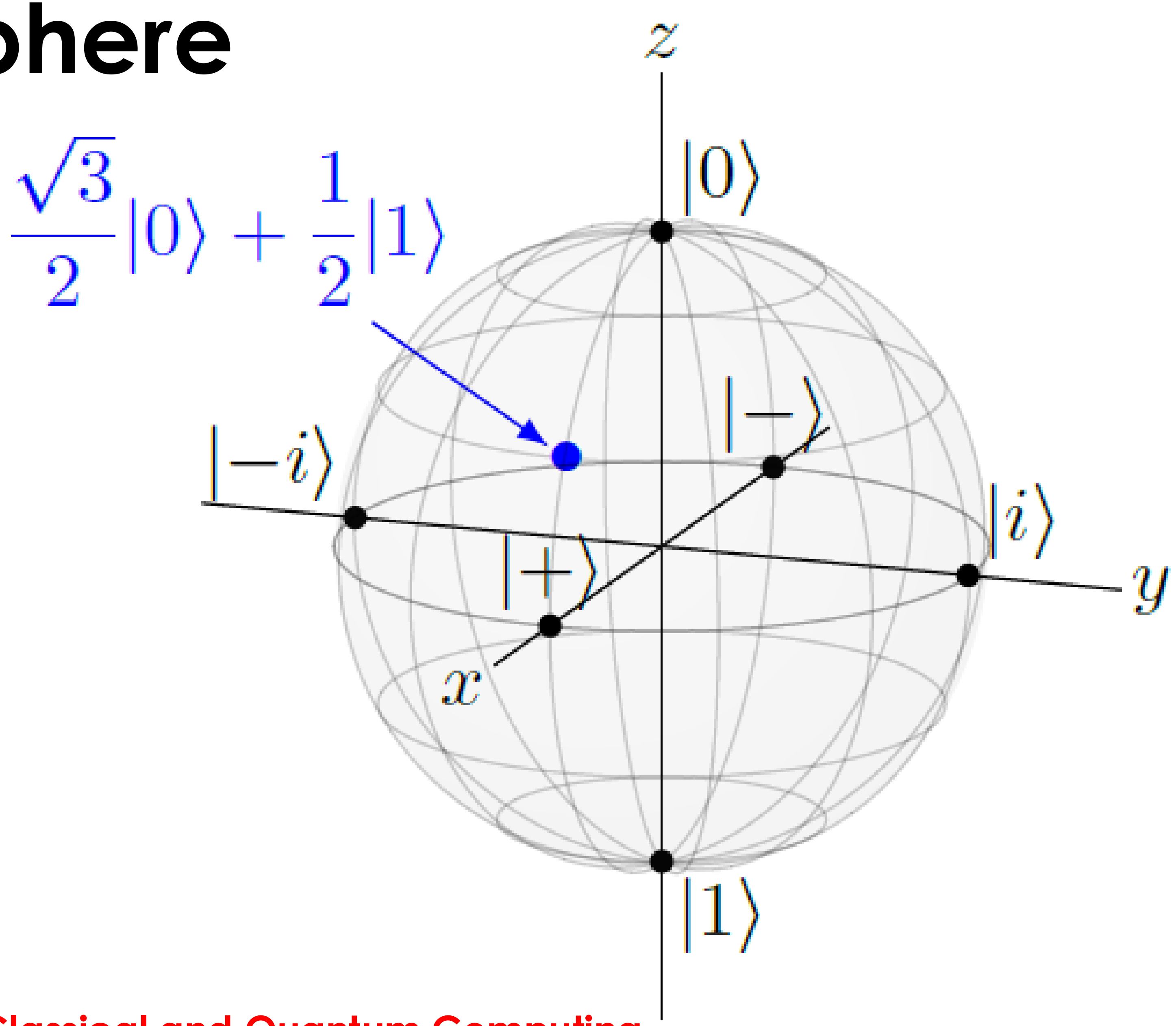
Bloch sphere



$|\Psi\rangle = |0\rangle$ $\quad$ $|\Psi\rangle = |1\rangle$ $\quad$ $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\quad$ $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

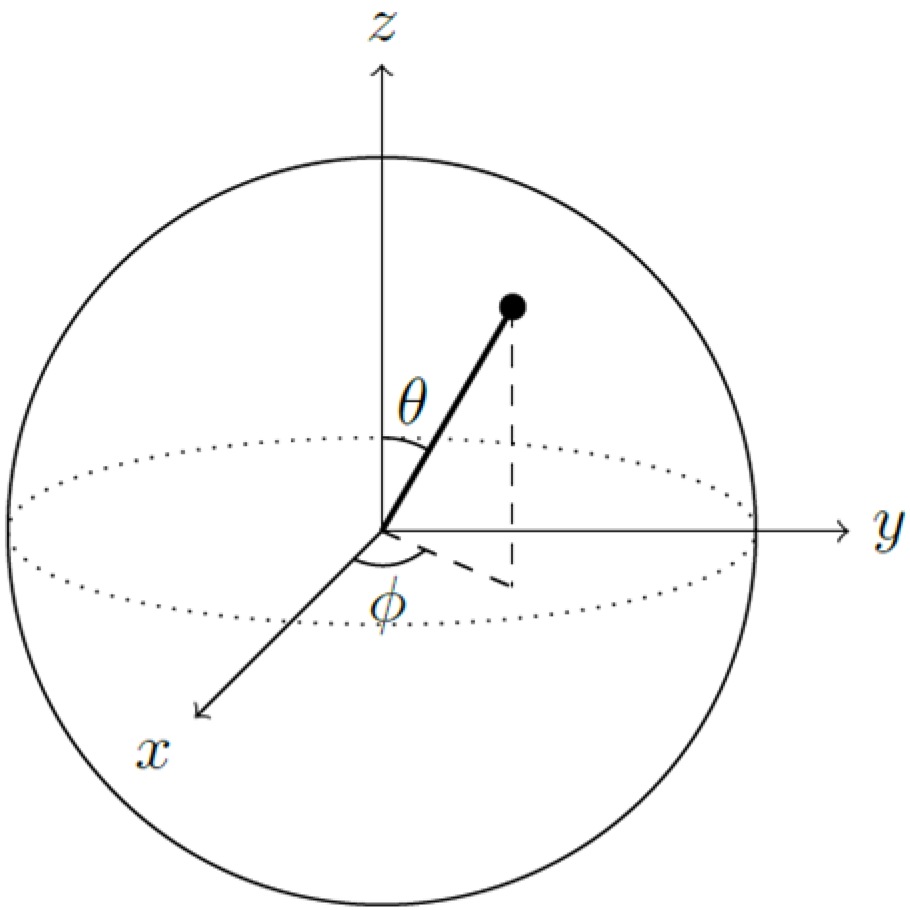
Book: Quantum Computing for the Quantum Curious

Bloch sphere



Book: Introduction to Classical and Quantum Computing

Bloch sphere



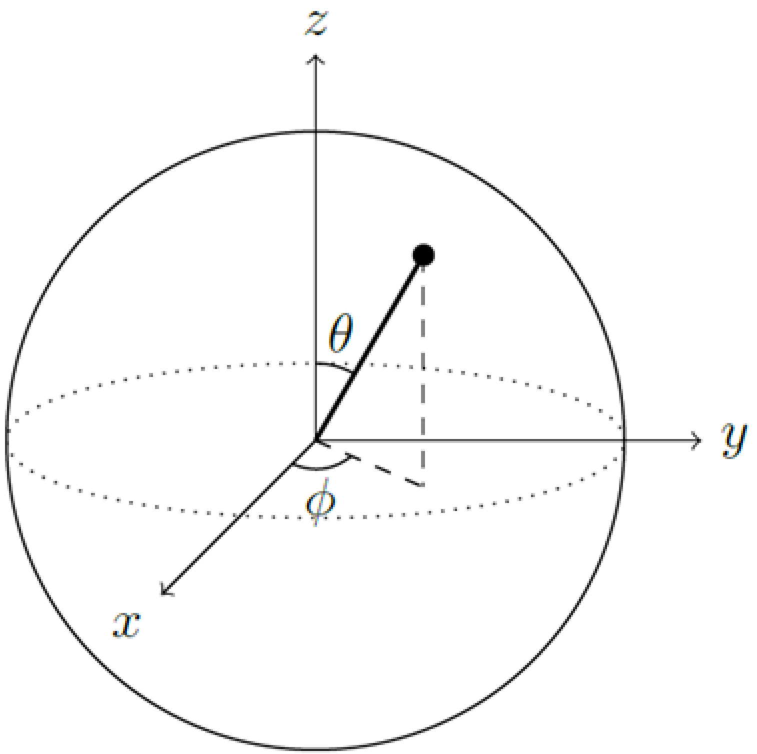
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

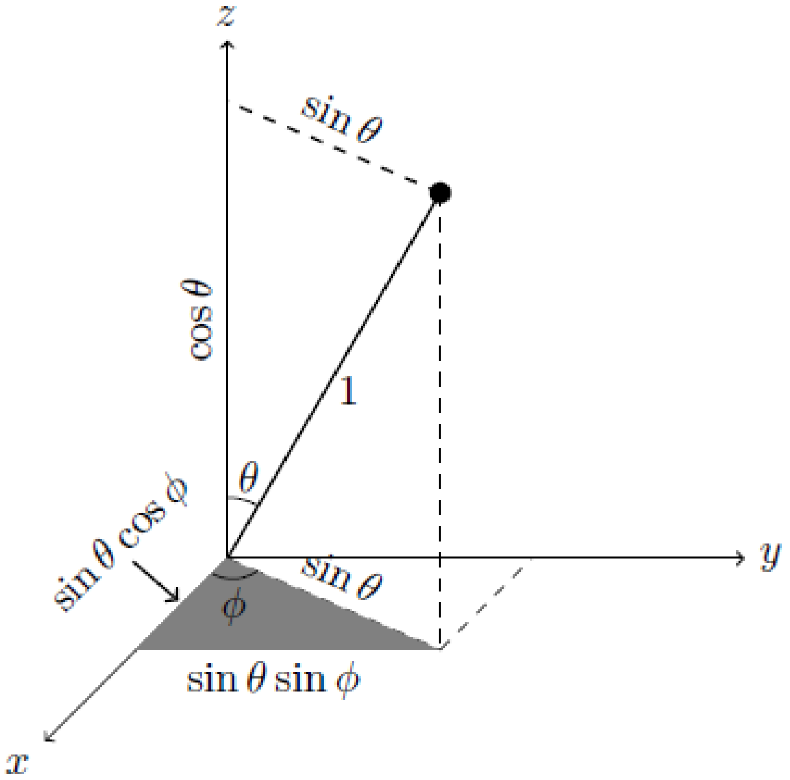
$$\alpha = \cos\left(\frac{\theta}{2}\right), \quad \beta = e^{i\phi}\sin\left(\frac{\theta}{2}\right)$$

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

Book: Introduction to Classical and Quantum Computing

Bloch sphere





$$x = \sin\theta\cos\phi,$$
$$y = \sin\theta\sin\phi,$$
$$z = \cos\theta.$$

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

Book: Introduction to Classical and Quantum Computing

Example

Consider the following state

$$|a\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle$$

Where on the Bloch sphere is this state?

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

$$\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{3}}{2} \qquad \frac{\theta}{2} = \frac{\pi}{6} \qquad \theta = \frac{\pi}{3}$$

$$e^{i\phi} = \cos(\phi) + i\sin(\phi)$$

$$i = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right) \qquad \phi = \frac{\pi}{2}$$

Book: Introduction to Classical and Quantum Computing

Example

Consider the following state

$$|a\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle$$

Where on the Bloch sphere is this state?

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

$$\theta = \frac{\pi}{3} \qquad \phi = \frac{\pi}{2}$$

$$x = \sin\theta\cos\phi,$$
$$y = \sin\theta\sin\phi,$$
$$z = \cos\theta.$$

$$x = 0 \quad y = \frac{\sqrt{3}}{2} \quad z = \frac{1}{2}$$

Book: Introduction to Classical and Quantum Computing

Check Your Understanding

Consider the following state

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\pi/6}|1\rangle\right)$$

Where on the Bloch sphere is this state?

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

$$x = \sin\theta\cos\phi,$$
$$y = \sin\theta\sin\phi,$$
$$z = \cos\theta.$$

Representing multiple qubits

Tensor product

$$\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} \otimes \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} \\ x_1 \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \otimes \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \begin{pmatrix} 3 \\ 4 \end{pmatrix} \\ 2 \begin{pmatrix} 3 \\ 4 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Representing multiple qubits

$$|00\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$|01\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$|10\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$|11\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

The product state of $\boldsymbol{n}$ bits is a vector of size $\boldsymbol{2^n}$

Operation on multiple qubits: CNOT

- Operators on pairs of qubits, one of which is the **"control"** qubits and the other the **"target"** qubit.
- If the **control qubit is 1**, then the target qubit is **flipped**.
- If the **control qubit is 0,** then the target qubit is **unchanged**.
- The control qubit is always unchanged.

$$|00\rangle \longrightarrow |00\rangle$$

$$|01\rangle \longrightarrow |01\rangle$$

$$|10\rangle \longrightarrow |11\rangle$$

$$|11\rangle \longrightarrow |10\rangle$$

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Operation on multiple qubits: CNOT

$$|00\rangle \longrightarrow |00\rangle \qquad |10\rangle \nearrow\!\!\!\!\!\searrow |10\rangle$$

$$|01\rangle \longrightarrow |01\rangle \qquad |11\rangle \qquad\quad |11\rangle$$

$$\text{CNOT}|00\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{CNOT}|01\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{CNOT}|00\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{CNOT}|01\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Superposition: the Hadamard gate

The Hadamard gate takes a 0 or 1 qubit and puts it into exactly equal superposition

$$\mathrm{H}|0\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad \mathrm{H}|1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Common One-Qubit Quantum Gates

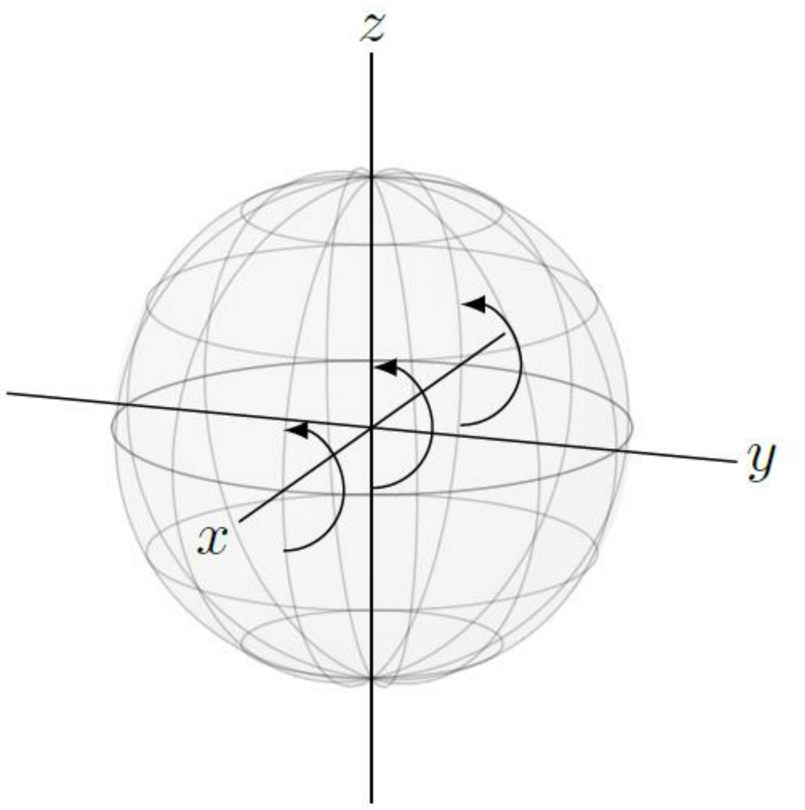
The *identity gate* turns $|0\rangle$ into $|0\rangle$ and $|1\rangle$ into $|1\rangle$, hence doing nothing:

$$I|0\rangle = |0\rangle,$$
$$I|1\rangle = |1\rangle.$$

Common One-Qubit Quantum Gates

The *Pauli X gate*, or *NOT gate*, turns $|0\rangle$ into $|1\rangle$, and $|1\rangle$ into $|0\rangle$:

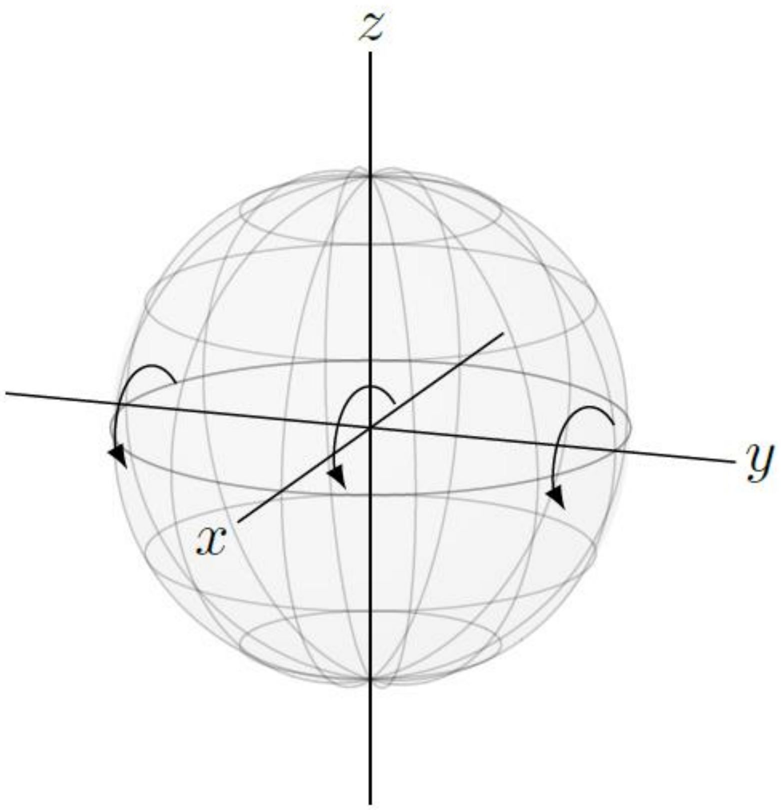
$$X|0\rangle = |1\rangle,$$
$$X|1\rangle = |0\rangle.$$



Common One-Qubit Quantum Gates

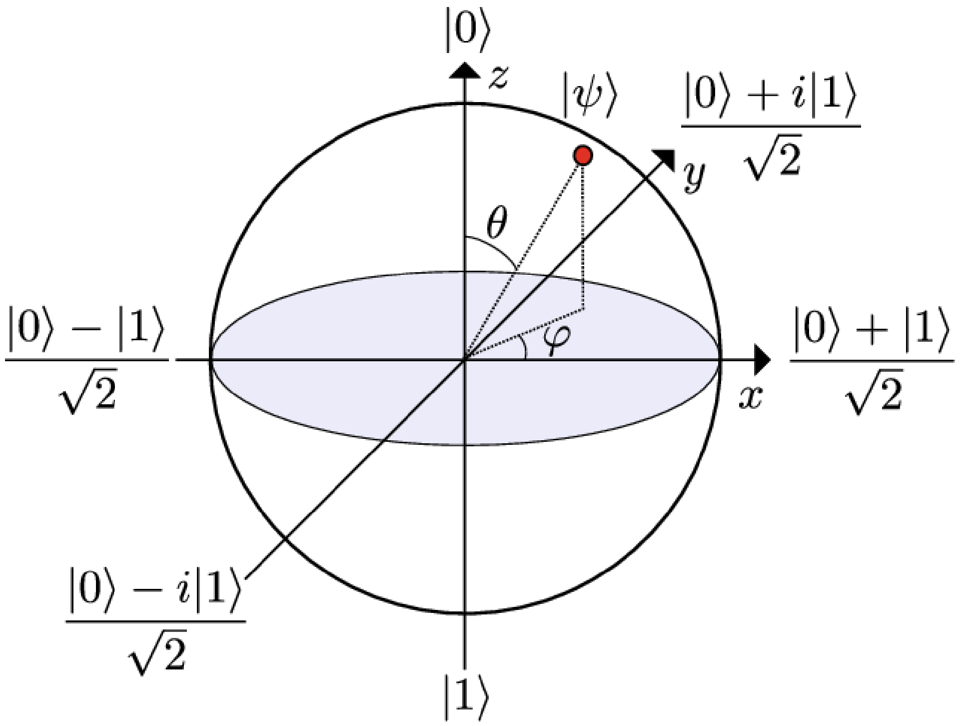
The *Pauli Y gate* turns $|0\rangle$ into $i|1\rangle$, and $|1\rangle$ into $-i|0\rangle$:

$$Y|0\rangle = i|1\rangle,$$
$$Y|1\rangle = -i|0\rangle.$$



Common One-Qubit Quantum Gates

The *Pauli Y gate* turns $|0\rangle$ into $i|1\rangle$, and $|1\rangle$ into $-i|0\rangle$:

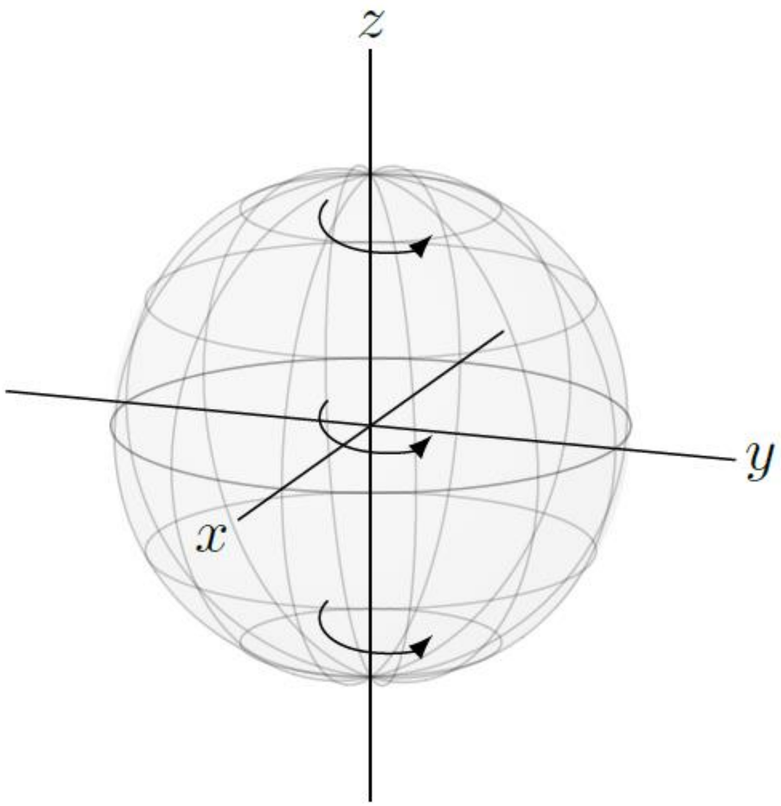


$$Y|0\rangle = i|1\rangle,$$
$$Y|1\rangle = -i|0\rangle.$$

Common One-Qubit Quantum Gates

The *Pauli Z gate* keeps $|0\rangle$ as $|0\rangle$ and turns $|1\rangle$ into $-|1\rangle$:

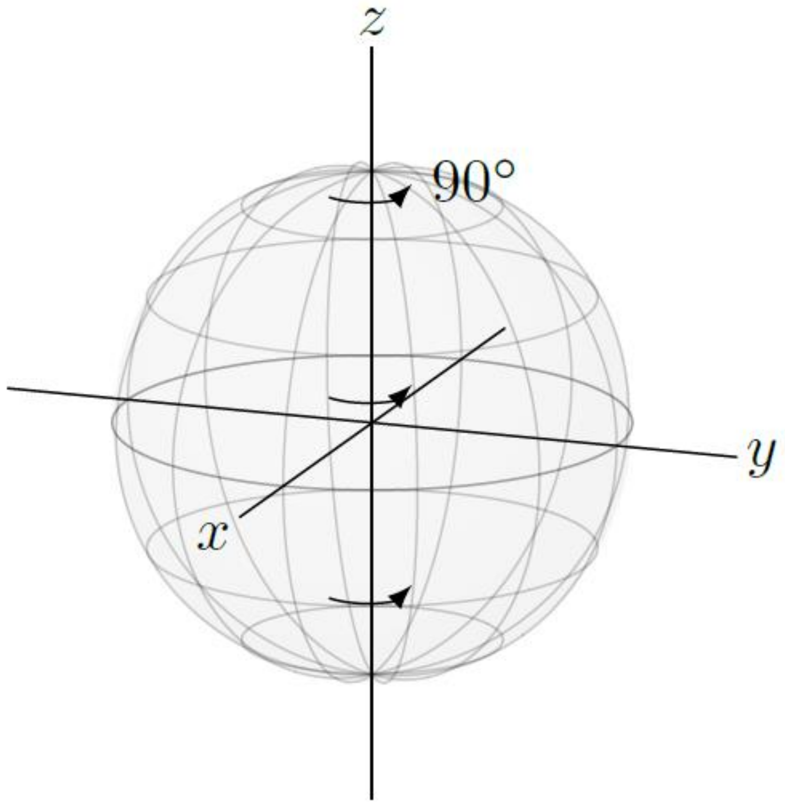
$$Z|0\rangle = |0\rangle,$$
$$Z|1\rangle = -|1\rangle.$$



Common One-Qubit Quantum Gates

Phase gate, which is the square root of the Z gate (i.e., $S^2 = Z$):

$$S|0\rangle = |0\rangle,$$
$$S|1\rangle = i|1\rangle.$$



Quantum circuit